PROPOSITIONAL LOGIC

- Kelvin Asclepius Minor –

1. Proposition is a declarative statement that is either True (T) or False (F). Propositions are often represented by variables (but not contain variables) to simplify the manipulation and analysis of logical statements. Decide whether the following statements are propositions !

   Every cow has 4 legs ; $3 + 2 = 32$; The sky is blue ; Do you like cake? ; Bring me coffee ; $x - 4 = 5$; $score \geq 90$; If you study, I will go home

2. Negation operator ($\sim$, $\neg$, or !) is one of the logical operators in propositional logic that inverts the truth value of a given proposition. Construct the truth table for Negation operator !

3. Negations can be formed by placing the word "not" in the appropriate place within the original statement, or by otherwise indicating that the original statement is false. Convert the following statements into their negations !

   It is raining ; The light is on ; The user is logged in ; The solution exists ; 3 is a positive number ; They are children

4. In Propositional Logic, the Double Negation Law occurs when negating a proposition twice, resulting in the original proposition itself. Write the expression of the Double Negation Law !

5. Compound proposition is a proposition that is formed by combining propositions using logical connectives (or logical operators) such as Conjunction ($\wedge$) and Disjunction ($\vee$). Conjunction operator joins two propositions, while Disjunction means choosing one of the propositions (possible to choose both). Construct the truth tables for Conjunction and Disjunction operators !

6. Convert the following compound propositions into their corresponding mathematical expressions using logical notation !

   I learn Mathematics and you learn Computer ; Mary bought apple and banana ; I will go to the party and see my friends ; The restaurant serves pizza or pasta ; He is intelligent, but he lacks experience ; You have a car or a house

7. The Idempotent Law occurs when a proposition applied multiple times using Conjunction and Disjunction operators and resulting the same proposition. Write the expression of the Idempotent Law !

8. Propositions can be structured using Commutative, Associative, and Distributive Laws with Conjunction and Disjunction operators. Write the expression of the Commutative, Associative, and Distributive Laws !

PROPOSITIONAL LOGIC

- Kelvin Asclepius Minor –

9. The Absorption Law states that for any propositions p and q, both $p \wedge (p \vee q)$ and $p \vee (p \wedge q)$ are logically equivalent to p. Show that the propositions are equivalent !
10. De Morgan's Laws describe how negations interact with Conjunction and Disjunction operations. Write the equivalences of Negation of Conjunction and Negation of Disjunction !

11. Disjunction operator ($\vee$) is often referred to as Inclusive (or Non-Exclusive) Disjunction, that means it is true if at least one of the propositions is true. In other hand, Exclusive Disjunction operator ($\oplus$) is true only if exactly one of the propositions is true, but not both. Exclusive Disjunction excludes the possibility of both propositions being true simultaneously. Construct the truth table for Exclusive Disjunction operator !
12. The Exclusive Disjunction statement captures the scenario where one proposition is true and the other is false. Write the equivalence of the Disjunction operator using Negation, Conjunction, and Disjunction operators !

13. Conditional (Implication) statement indicates that proposition p is a sufficient condition for proposition q as the necessary condition, and it can be expressed in the form $p \Rightarrow q$. This means that if p is true, it is enough to guarantee that q will be true, but it does not mean that q can only be true if p is true. Construct the truth table for Conditional operator !
14. Given the propositions p: $study\ hard$ and q: $get\ an\ A$, write the proposition that express $p \Rightarrow q$ using various grammatical constructions !
15. For Conditional statement $p \Rightarrow q$, find the position of propositions p and q in the following statements !

    If there is sunlight, then the photosynthesis occurs ; You will get a discount if you buy more than ten items ; You must show your ID whenever you enter the building ; The project will start, provided that the funding is approved ; Whenever there is a blackout, the generator will turn on; Being a member of the club is a sufficient condition for accessing the lounge ; For the cake to bake properly, it is sufficient that the oven is preheated ; Having the correct password is a necessary condition for logging in; For it to have rained, it is necessary that the ground is wet ; It rains only if the ground is wet

16. The Conditional statement $p \Rightarrow q$ captures the scenario when the proposition p is false or the proposition q is true $(\neg p \vee q)$. Show that the propositions are equivalent !
17. Write the equivalence of the propositions $p \Rightarrow (q \wedge r)$, $p \Rightarrow (q \vee r)$, $(p \wedge q) \Rightarrow r$, and $(p \vee q) \Rightarrow r$!
18. The Converse of Conditional statement $p \Rightarrow q$ is formed by reversing the positions of p and q. Write the Converse using the Conditional statement and construct the truth table for the Converse of Conditional statement !
19. The Inverse of Conditional statement $p \Rightarrow q$ is formed by negating p and q. Write the Inverse using the Conditional statement and construct the truth table for the Inverse of Conditional statement !
20. The Contrapositive of Conditional statement $p \Rightarrow q$ is formed by reversing and negating both p and q. Write the Inverse using the Conditional statement and construct the truth table for the Contrapositive of Conditional statement !

21. Biconditional (Bi-implication) statement indicates that two propositions p and q are logically equivalent, and it can be expressed in the form $p \Leftrightarrow q$. This means that $p \Leftrightarrow q$ is true if and only if p and q have the same truth value. Construct the truth table for Biconditional operator !
22. Given the propositions p: $have\ ability\ to\ breathe$ and q: $have\ life$, write the proposition that express $p \Leftrightarrow q$ using various grammatical constructions !
23. The Biconditional statement $p \Leftrightarrow q$ captures the scenario when the proposition p is the sufficient condition for proposition q and proposition q is the sufficient condition for proposition p. Show that the propositions are equivalent !
24. The Biconditional statement $p \Leftrightarrow q$ captures the scenario when the negation of proposition p is the sufficient condition for the negation of proposition q and the negation of proposition q is the sufficient condition for the negation of proposition p. Show that the propositions are equivalent !
25. The Biconditional statement $p \Leftrightarrow q$ captures the scenario when the propositions p and q are both true or the propositions p and q are both false. Show that the propositions are equivalent !

PROPOSITIONAL LOGIC

- Kelvin Asclepius Minor –

26. In Propositional Logic, two fundamental concepts related to the truth values of propositions are Tautology and Contradiction. Tautology is a propositional that is always True under every possible interpretation of truth values to its variables, while Contradiction is a propositional that is always False under every possible interpretation of truth values to its variables. Write the Negation Laws by determining statements $p \wedge \neg p$ and $p \vee \neg p$ are Tautology or Contradiction !
27. Write the Domination Laws by analyzing if a proposition combined with a Contradiction using Conjunction and with a Tautology using Disjunction !
28. Write the Identity Laws by analyzing if a proposition combined with a Tautology using Conjunction and with a Contradiction using Disjunction !

29. Using the Logical Equivalence Laws, find the equivalence for the negation of Conditional statement $p \Rightarrow q$!
30. Using the Logical Equivalence Laws, find the equivalence for the negation of Biconditional statement $p \Leftrightarrow q$!

31. Show that $p \vee \neg q \implies \neg p \wedge \neg q$ and $\neg p$ are equivalent !
32. Show that p and $(\neg p \vee \neg q) \wedge \neg p$ are equivalent !
33. Show that $p \vee \neg p \wedge q$ and $p \vee q$ are equivalent !
34. Show that $((p \vee \neg q) \wedge (q \vee r)) \wedge q$ and $p \wedge q$ are equivalent !
35. Show that $\neg p \wedge \neg r \vee p \wedge \neg q \wedge \neg r$ and $\neg r \wedge (q \Rightarrow \neg(p \wedge \neg r))$ are equivalent !

36. Show that $(p \vee q) \wedge (p \vee \neg q)$ is a Tautology !
37. Show that $\neg((\neg p \vee \neg q) \wedge q) \vee \neg p$ is a Tautology !
38. Show that $p \implies p \vee q$ is a Tautology !
39. Show that $p \wedge q \implies p$ is a Tautology !
40. Show that $(p \vee q) \wedge \neg q \implies p$ is a Tautology !

41. In Propositional Logic, an Argument is a sequence of statements or propositions, where some of these statements (called Premises) are intended to support or provide evidence for another statement (called the Conclusion). A valid Argument can be considered equivalent to a Tautology when expressed in the form of an Implication $[premise_1 \wedge premise_2 \wedge premise_3 \wedge \ldots \wedge premise_n] \Longrightarrow conclusion$ and gives True value in all possible truth assignments. Modus Ponens is a form of argument in propositional logic that involves two premises $p \Longrightarrow q$ and p that leads to a conclusion q. Show the validity of the argument !
42. Modus Tollens is a form of argument in propositional logic that involves two premises $p \Longrightarrow q$ and $\neg q$ that leads to a conclusion $\neg p$. Show the validity of the argument !
43. Transitive Syllogism (or Hypothetical Syllogism) is a form of argument in propositional logic that involves two premises $p \Longrightarrow q$ and $q \Longrightarrow r$ that leads to a conclusion $p \Longrightarrow r$. Show the validity of the argument !
44. Disjunctive Syllogism is a form of argument in propositional logic that involves two premises $p \vee q$ and $\neg p$ that leads to a conclusion q. Show the validity of the argument !
45. Resolution is a form of argument in propositional logic that involves two premises $p \vee q$ and $\neg p \vee r$ that leads to a conclusion $q \vee r$. Show the validity of the argument !
46. In Propositional Logic, Addition is one of the Rules of Inference that allows inferring a disjunction from a given statement p with any statement q, regardless of the truth value of q. Show the validity of the argument!
47. In Propositional Logic, Simplification is one of the Rules of Inference that allows inferring of the conjuncts from a conjunction $p \wedge q$, giving the conclusion p or q. Show the validity of the argument !
48. In Propositional Logic, Conjunction is one of the Rules of Inference that allows combine two true statements p and q into a single conjunction $p \wedge q$. Show the validity of the argument !
49. Given 4 premises $\neg p \wedge q$, $r \Longrightarrow p$, $\neg r \Longrightarrow s$, and $s \Longrightarrow t$. Show that t is a conclusion !
50. Given 5 premises $\neg p \vee q \Longrightarrow r$, $s \vee \neg q$, $\neg t$, $p \Longrightarrow t$, and $\neg p \wedge r \Longrightarrow \neg s$. Show that $\neg q$ is a conclusion !

51. If Adam is at the scene of the crime, then Adam has a motive. There is either a witness who saw Adam or there is usable DNA evidence. If there is no DNA evidence, then there is no evidence. Adam does not have a motive or there is usable DNA evidence. After observation, there is no usable DNA evidence. If Adam is not at the scene of the crime and there is a witness who saw Adam, then Adam is not guilty. There are no fingerprints at the scene of the crime or there is evidence at the scene of the crime. Please help Adam to prove he is not guilty and there is no evidence at the scene !
52. If Richard goes to class by walking, then Richard walks with me. Richard either buys a shirt the same color as me or Richard becomes a member of my group for the project. If Richard does not become a member of my group for the project, then Richard does not want to get a bad grade. Richard does not walk with me, or Richard wants to be late to class. On that day, Richard does not want to be late, and he does not buy a shirt the same color as me. If Richard does not go to class by walking even though Richard becomes a member of my group for the project, then Richard does not love me. Please help me to verify if Richard really loves me !