FUZZY SET THEORY

- Kelvin Asclepius Minor –

1. The Fuzzy Set Theory was introduced by Lotfi A. Zadeh in 1965 to represent uncertainty and imprecision in a Set. Unlike the classical Set Theory, where an element either belongs to the set or does not belong to the set, the Fuzzy Set Theory allows for degrees of membership that range between 0 and 1. A Fuzzy Set A is defined within a universal set X. In a Fuzzy Set A, each element $x \in X$ has a degree of membership represented by a membership function $\mu_A(x)$. Therefore, A Fuzzy Set A can be expressed as a set of ordered pairs $A = \{(x, \mu_A(x)) | x \in X\}$. Consider five students: Adrian, Bryan, Cindy, Damian, and Erick. Adrian and Cindy like studying very much; they study every day. Bryan also likes studying, but he only studies four days a week. Erick doesn't like studying; he only studies when there is an exam. On the other hand, Damian never studies, even when there is an exam. Define a Fuzzy Set to represent these students !
2. For temperatures below $10°C$, the comfort degree is low. For temperatures between $10°C$ and $20°C$, the comfort degree increases linearly. For temperatures between $20°C$ and $30°C$, the comfort degree remains high. For temperatures between $30°C$ and $40°C$, the comfort degree decreases linearly. Define a Fuzzy Set to represent these degree of comforts !

3. In Fuzzy Set Theory, the Complement of a Fuzzy Set represents the degree to which elements do not belong to the Fuzzy Set. Given a Fuzzy Set A with a membership function $\mu_A(x)$, the membership function of the Complement Fuzzy Set $\bar{A}$ (or A^C) can be defined as $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$. If Fuzzy Set $A = \{(P, 0.1), (Q, 0.3), (R, 0.7), (S, 1.0), (T, 0.6), (U, 0.2), (V, 0.0)\}$, define Fuzzy Set $\bar{A}$!
4. In Fuzzy Set Theory, the Support of a Fuzzy Set describes the range of elements in the domain that have a non-zero degree of membership in the Fuzzy Set. Given a Fuzzy Set A with a membership function $\mu_A(x)$, then the Support of the Fuzzy Set can be defined as $supp(A) = \{x \in X | \mu_A(x) > 0\}$. If Fuzzy Set $A = \{(P, 0.1), (Q, 0.3), (R, 0.7), (S, 1.0), (T, 0.6), (U, 0.2), (V, 0.0)\}$, define $supp(A)$!
5. In Fuzzy Set Theory, the Alpha-Cut ($\alpha - cut$) of a Fuzzy Set describes the range of elements in the domain that have membership values greater than or equal to a certain threshold α. Given a Fuzzy Set A with a membership function $\mu_A(x)$, then the Alpha-Cut of the Fuzzy Set can be defined as $A_\alpha = \{x \in X | \mu_A(x) \geq \alpha\}$. If Fuzzy Set $A = \{(P, 0.1), (Q, 0.3), (R, 0.7), (S, 1.0), (T, 0.6), (U, 0.2), (V, 0.0)\}$ define $A_{0.3}$!

6. In Fuzzy Set Theory, the Cardinality of a Fuzzy Set describes the size of the Fuzzy Set considering the degrees of membership of its elements. Given a Fuzzy Set A with a membership function $\mu_A(x)$, then the Cardinality of the Fuzzy Set can be defined as $|A| = \sum_{x \in X} \mu_A(x)$. If Fuzzy Set $A = \{(P, 0.1), (Q, 0.3), (R, 0.7), (S, 1.0), (T, 0.6), (U, 0.2), (V, 0.0)\}$define $|A|$!
7. In Fuzzy Set Theory, the Union of two Fuzzy Sets $A \cup B$ combines their membership functions to reflect the maximum degree of membership for each element, that can be defined as $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$. If Fuzzy Set $A = \{(P, 0.1), (Q, 0.3), (R, 0.7), (S, 1.0), (T, 0.6), (U, 0.2), (V, 0.0)\}$ and Fuzzy Set $B = \{(P, 0.2), (Q, 0.8), (R, 1.0), (S, 0.6), (T, 0.4), (U, 0.3), (V, 0.1)\}$, define Fuzzy Set $A \cup B$!
8. In Fuzzy Set Theory, the Intersection of two Fuzzy Sets $A \cap B$ combines their membership functions to reflect the minimum degree of membership for each element, that can be defined as $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$. If Fuzzy Set $A = \{(P, 0.1), (Q, 0.3), (R, 0.7), (S, 1.0), (T, 0.6), (U, 0.2), (V, 0.0)\}$ and Fuzzy Set $B = \{(P, 0.2), (Q, 0.8), (R, 1.0), (S, 0.6), (T, 0.4), (U, 0.3), (V, 0.1)\}$, define Fuzzy Set $A \cap B$!
9. In Fuzzy Set Theory, Fuzzy Set B is a Subset of Fuzzy Set A $(B \subseteq A)$ if $\mu_A(x) \geq \mu_B(x)$. If Fuzzy Set $A = \{(P, 0.1), (Q, 0.3), (R, 0.7), (S, 1.0), (T, 0.6), (U, 0.2), (V, 0.0)\}$ and Fuzzy Set $B = \{(P, 0.2), (Q, 0.8), (R, 1.0), (S, 0.6), (T, 0.4), (U, 0.3), (V, 0.1)\}$, determine whether A is a Subset of B !
10. If Fuzzy Set $A = \{(P, 0.1), (Q, 0.3), (R, 0.7), (S, 1.0), (T, 0.6), (U, 0.2), (V, 0.0)\}$ and Fuzzy Set $B = \{(P, 0.2), (Q, 0.8), (R, 1.0), (S, 0.6), (T, 0.4), (U, 0.3), (V, 0.1)\}$, define $supp(A \cap B)$, $(A \cup B)_{0.8}$, and$|A \cup B|$!

11. A If Fuzzy Set $A = \left\{ \left(x, \mu_A(x)\right) \middle| 0 \leq x \leq 12, \mu_A(x) = \begin{cases} 0 & ,when\ x \leq 4 \\ x - 4 & ,when\ 4 < x \leq 5 \\ 1 & ,when\ 5 < x \leq 8 \\ 5 - \frac{1}{2}x & ,when\ 8 < x \leq 10 \\ 0 & ,when\ x > 10 \end{cases} \right\}$, define Fuzzy Set $\bar{A}$, $A_{0.5}$, and $supp(A)$!
12. If Fuzzy Set $B = \left\{ \left(x, \mu_B(x)\right) \middle| 0 \leq x \leq 12, \mu_B(x) = \begin{cases} 0 & ,when\ x \leq 2 \\ \frac{x-2}{3} & ,when\ 2 < x \leq 5 \\ \frac{9-x}{4} & ,when\ 5 < x \leq 9 \\ 0 & ,when\ x > 9 \end{cases} \right\}$, define Fuzzy Set $\bar{B}$, $B_{0.6}$, and $supp(B)$!

13. If Fuzzy Set $A = \left\{ \left(x, \mu_A(x)\right) \middle| 0 \le x \le 12, \mu_A(x) = \begin{cases} 0 & , when\ x \le 4 \\ x - 4 & , when\ 4 < x \le 5 \\ 1 & , when\ 5 < x \le 8 \\ 5 - \frac{1}{2}x & , when\ 8 < x \le 10 \\ 0 & , when\ x > 10 \end{cases} \right\}$, and If Fuzzy Set

$B = \left\{ \left(x, \mu_B(x)\right) \middle| 0 \le x \le 12, \mu_B(x) = \begin{cases} 0 & , when\ x \le 2 \\ \frac{x-2}{3} & , when\ 2 < x \le 5 \\ \frac{9-x}{4} & , when\ 5 < x \le 9 \\ 0 & , when\ x > 9 \end{cases} \right\}$, define $A \cup B$ and $A \cap B$!

14. If Fuzzy Set $A = \left\{ \left(x, \mu_A(x)\right) \middle| 0 \le x \le 12, \mu_A(x) = \begin{cases} 0 & , when\ x \le 4 \\ x - 4 & , when\ 4 < x \le 5 \\ 1 & , when\ 5 < x \le 8 \\ 5 - \frac{1}{2}x & , when\ 8 < x \le 10 \\ 0 & , when\ x > 10 \end{cases} \right\}$, and If Fuzzy Set

$B = \left\{ \left(x, \mu_B(x)\right) \middle| 0 \le x \le 12, \mu_B(x) = \begin{cases} 0 & , when\ x \le 2 \\ \frac{x-2}{3} & , when\ 2 < x \le 5 \\ \frac{9-x}{4} & , when\ 5 < x \le 9 \\ 0 & , when\ x > 9 \end{cases} \right\}$, define $\overline{(A \cup B)}$, $\overline{(A \cap B)}$, $\bar{A} \cup \bar{B}$, and $\bar{A} \cap \bar{B}$!

15. If Fuzzy Set $A = \left\{ \left(x, \mu_A(x)\right) \middle| 0 \le x \le 12, \mu_A(x) = \begin{cases} 0 & , when\ x \le 4 \\ x - 4 & , when\ 4 < x \le 5 \\ 1 & , when\ 5 < x \le 8 \\ 5 - \frac{1}{2}x & , when\ 8 < x \le 10 \\ 0 & , when\ x > 10 \end{cases} \right\}$, and If Fuzzy Set

$B = \left\{ \left(x, \mu_B(x)\right) \middle| 0 \le x \le 12, \mu_B(x) = \begin{cases} 0 & , when\ x \le 2 \\ \frac{x-2}{3} & , when\ 2 < x \le 5 \\ \frac{9-x}{4} & , when\ 5 < x \le 9 \\ 0 & , when\ x > 9 \end{cases} \right\}$, define $supp(A \cup B)$ and $(A \cup B)_{0.35}$!